

Name:

Solutions / Answers

Without a calculator, evaluate each limit:

1. $\lim_{x \rightarrow 2} (x^2 + 3x - 1) = (2)^2 + 3(2) - 1 = 4 + 6 - 1 = 9$

2. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$

3. $\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$

4. $\lim_{t \rightarrow 3^+} \frac{2}{t-3} = \infty$

5. $\lim_{k \rightarrow 0} \frac{|k|}{k} = \text{undefined}$

$$6. \lim_{y \rightarrow 2} \frac{y^2 - 3y + 2}{y^2 - 4} = \lim_{y \rightarrow 2} \frac{(y-1)(y-2)}{(y-2)(y+2)} = \lim_{y \rightarrow 2} \frac{y-1}{y+2} = \frac{1}{4}$$

$$7. \lim_{t \rightarrow \infty} \frac{6t^2 + 5t}{2t^2 + 8} = \frac{6}{2} = 3$$

$$8. \lim_{t \rightarrow \infty} \frac{5t-3}{t^2-9} = 0$$

$$9. \lim_{t \rightarrow \infty} \frac{6t^3 + 5t^2}{2t^2 - t} = \infty$$

$$10. \text{ Given } f(t) = \begin{cases} 2t+4 & \text{if } t \leq -1 \\ -3t-2 & \text{if } t > -1 \end{cases}, \quad \lim_{t \rightarrow -1^+} f(t) = 1$$

11. Given $f(t) = \begin{cases} 2t+4 & \text{if } t \leq -1 \\ -3t-2 & \text{if } t > -1 \end{cases}$, $\lim_{t \rightarrow -1} f(t) =$ undefined since

$$\lim_{t \rightarrow -1^-} f(t) \neq \lim_{t \rightarrow -1^+} f(t)$$

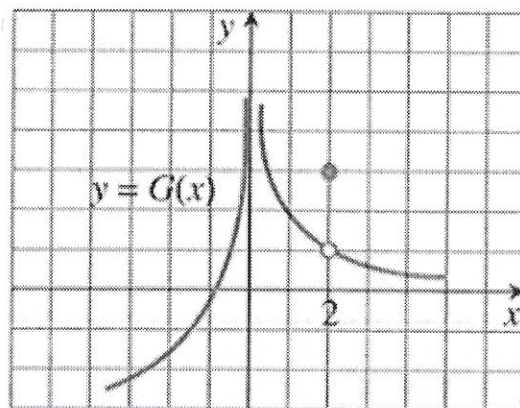
$$2 \neq 1$$

12. Given $g(x) = \begin{cases} 5-x & \text{if } x < 2 \\ \frac{1}{2}x+2 & \text{if } x \geq 2 \end{cases}$, $\lim_{x \rightarrow 2} g(x) = 3$ since $\lim_{x \rightarrow 2^+} g(x) = 3$ and $\lim_{x \rightarrow 2^-} g(x) = 3$

13. Given the graph of $y = G(x)$, determine each of the following:

a. $G(2) = 3$

b. $\lim_{x \rightarrow 2} G(x) = 1$



14. Evaluate $\lim_{x \rightarrow \infty} \frac{5x + \sin x}{x} = \lim_{x \rightarrow \infty} \left(\frac{5x}{x} + \frac{\sin x}{x} \right) = \lim_{x \rightarrow \infty} (5) + \lim_{x \rightarrow \infty} \frac{\sin x}{x}$

$$= 5 + 0$$

$$= 5$$

15. Evaluate $\lim_{x \rightarrow 0} \frac{5x + \sin x}{x} = \lim_{x \rightarrow 0} \left(\frac{5x}{x} + \frac{\sin x}{x} \right) = \lim_{x \rightarrow 0} (5) + \lim_{x \rightarrow 0} \frac{\sin x}{x}$
 $= 5 + 1$
 $= 6$

16. Give the equations of the two vertical asymptotes closest to the origin on the graph of $f(x) = \tan x$

$$x = -\frac{\pi}{2} \quad \text{and} \quad x = \frac{\pi}{2}$$

17. Find a simple right end behavior model for the function $f(x) = e^x - 2x$

$$g(x) = e^x$$

18. Evaluate $\lim_{k \rightarrow -\infty} \frac{1 - \cos x}{x^2} = 0$

19. Evaluate $\lim_{x \rightarrow \infty} \left(\frac{2}{x} + 1 \right) \left(\frac{5 + x^2}{x^2} \right) = \lim_{x \rightarrow \infty} \left(\frac{2}{x} + 1 \right) \cdot \lim_{x \rightarrow \infty} \left(\frac{5}{x^2} + 1 \right)$
 $= (1) \cdot (1)$
 $= 1$

20. Write a power function end behavior model for the function $f(x) = 3x^2 + 2x - 1$

$$g(x) = 3x^2$$

21. Write a power function end behavior model for the function $f(x) = \frac{x^5 - x^4 + x + 1}{2x^2 + x - 3}$

$$g(x) = \frac{1}{2}x^3$$

22. Write a power function end behavior model for the function $f(x) = \frac{x-2}{2x^2+3x-5}$

$$g(x) = \frac{1}{2x}$$

23. Evaluate $\lim_{x \rightarrow \infty} (x^2 e^{-x}) = \lim_{x \rightarrow \infty} \left(\frac{x^2}{e^x} \right) = 0$

24. Evaluate $\lim_{x \rightarrow -\infty} (x^2 e^{-x}) = \infty$

25. Evaluate $\lim_{x \rightarrow 3} \frac{x-3}{x^3-27} = \lim_{x \rightarrow 3} \frac{x-3}{(x-3)(x^2+3x+9)} = \lim_{x \rightarrow 3} \frac{1}{x^2+3x+9}$

$$= \frac{1}{(3)^2+3(3)+9}$$

$$= \frac{1}{27}$$

26. Sketch a graph of a function $y = f(x)$ that satisfies the stated conditions. Include any asymptotes.

Conditions: $\lim_{x \rightarrow 1} f(x) = 2$, $\lim_{x \rightarrow 5^-} f(x) = \infty$, $\lim_{x \rightarrow 5^+} f(x) = \infty$, $\lim_{x \rightarrow \infty} f(x) = -1$,

$\lim_{x \rightarrow -2^+} f(x) = -\infty$, $\lim_{x \rightarrow -2^-} f(x) = \infty$, & $\lim_{x \rightarrow -\infty} f(x) = 0$

